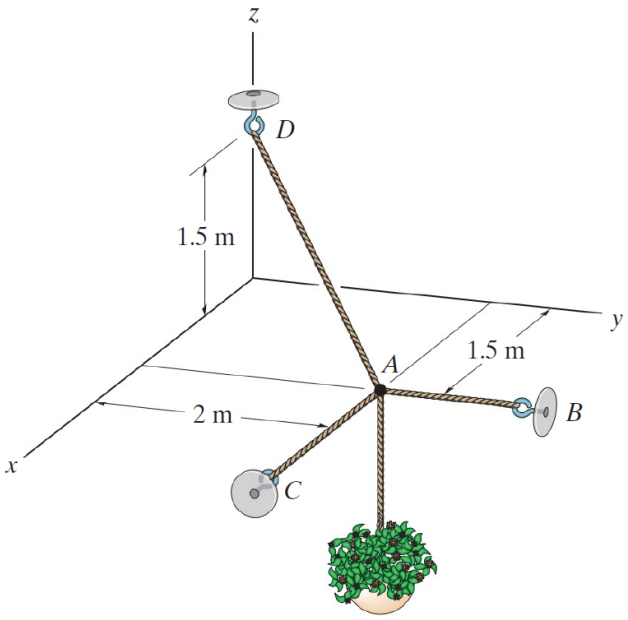
Problem 3-43

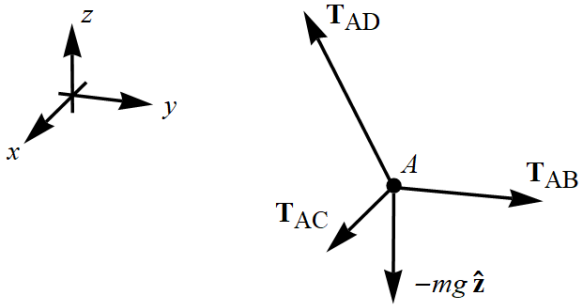
The three cables are used to support the 40-kg flowerpot. Determine the force developed in each cable for equilibrium.



Prob. 3–43

Solution

Draw a free-body diagram for the point at A.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + (-mg\hat{\mathbf{z}}) = \mathbf{0}$$

The position vector to point A is $\langle 1.5, 2, 0\rangle$ m, and the position vector to point D is $\langle 0, 0, 1.5\rangle$ m.

$$T_{AB}\hat{\mathbf{y}} + T_{AC}\hat{\mathbf{x}} + T_{AD}\hat{\mathbf{u}}_{AD} + (-mg\hat{\mathbf{z}}) = \mathbf{0}$$

$$T_{AB}\langle 0,1,0\rangle + T_{AC}\langle 1,0,0\rangle + T_{AD}\frac{\langle 0-1.5, 0-2, 1.5-0\rangle}{\sqrt{(0-1.5)^2+(0-2)^2+(1.5-0)^2}} + \langle 0,0,-mg\rangle = \mathbf{0}$$

Simplify the left side.

$$T_{\text{AB}}\langle 0,1,0\rangle + T_{\text{AC}}\langle 1,0,0\rangle + T_{\text{AD}}\left\langle -\frac{3}{\sqrt{34}}, -2\sqrt{\frac{2}{17}}, \frac{3}{\sqrt{34}}\right\rangle + \langle 0,0,-mg\rangle = \mathbf{0}$$

$$\left\langle T_{\text{AC}} - \frac{3}{\sqrt{34}}T_{\text{AD}}, T_{\text{AB}} - 2\sqrt{\frac{2}{17}}\,T_{\text{AD}}, \frac{3}{\sqrt{34}}T_{\text{AD}} - mg\right\rangle = \langle 0,0,0\rangle$$

Match the components on both sides to get a system of equations.

$$\left.\begin{aligned} T_{\text{AC}} - \frac{3}{\sqrt{34}}T_{\text{AD}} &= 0 \\ T_{\text{AB}} - 2\sqrt{\frac{2}{17}}\,T_{\text{AD}} &= 0 \\ \frac{3}{\sqrt{34}}T_{\text{AD}} - mg &= 0 \end{aligned}\right\}$$

Solving it yields

$$T_{\text{AB}} = \frac{4mg}{3} \quad \text{and} \quad T_{\text{AC}} = mg \quad \text{and} \quad T_{\text{AD}} = \frac{mg\sqrt{34}}{3}.$$

Therefore, since $m = 40$ kg and $g = 9.81$ m/s^2,

$$T_{\text{AB}} \approx 523 \text{ N} \quad \text{and} \quad T_{\text{AC}} \approx 392 \text{ N} \quad \text{and} \quad T_{\text{AD}} \approx 763 \text{ N}.$$